

Phase transition in the fine structure constant

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Within the context of mass-varying neutrinos, we construct a cosmological model that has a phase transition in the electromagnetic fine structure constant α at a redshift of 0.5. The model accommodates hints of a time variable α in quasar spectra and the nonobservance of such an effect at very low redshifts. It is consistent with limits from the recombination and primordial nucleosynthesis eras and is free of instabilities.

I. INTRODUCTION

Measurements of the cosmic microwave background, large scale structure, the evolution of the Hubble parameter from luminosity-redshift relation of type Ia supernovae along with the abundances of light elements in the universe strongly indicate the existence of a dark energy of unknown origin that acts against the pull of gravity [1]. The combined data favor an effective de-Sitter constant that nearly saturates the upper bound given by the present-day value (which we denote by a subscript 0 to indicate redshift $z = 0$) of the Hubble parameter $H_0 \approx 10^{-33}$ eV. This yields a dark energy density: $\rho_{\text{DE}} \sim 3M_{\text{Pl}}^2 H_0^2 \sim (2.4 \times 10^{-3} \text{ eV})^4$, where $M_{\text{Pl}} \simeq 2.4 \times 10^{18}$ GeV is the Planck mass.

The coincidence of the neutrino mass scale with the dark energy mass scale is suggestive that there may be a link between these quantities. Measurements of atmospheric neutrinos have provided evidence (at $> 15\sigma$) for ν_μ disappearing (likely converting to ν_τ) when propagating over distances of order hundreds (or more) kilometers. The corresponding oscillation phase is consistent with being maximal and the oscillations require a neutrino mass-squared difference of $\delta m_{\text{atm}}^2 \sim 2.5 \times 10^{-3} \text{ eV}^2$ [2]. The ν_μ disappearance oscillations have been confirmed by the KEK-to-Kamioka (K2K) and MINOS experiments over baselines of 250 km and 730 km, respectively. To implement a connection of dark energy and neutrino mass [3] in a concrete manner, Fardon, Nelson, and Weiner (FNW) [4] introduced a Yukawa coupling between a sterile neutrino and a cosmic scalar field (dubbed the accelaron), such that the neutrino masses m_{ν_i} are generated by the vacuum expectation value \mathcal{A} of this field, *i.e.*, $m_{\nu_i}(\mathcal{A})$. For simplicity hereafter we only consider a single nonvanishing neutrino mass, m_ν . The active neutrino mass is determined through a seesaw mechanism by integrating out the heavy sterile neutrino with mass $M(\mathcal{A})$, now correlated with the accelaron. This gives an effective potential

$$\begin{aligned} V_{\text{eff}}^{\text{NR}} &= m_\nu(\mathcal{A}) n_\nu + V[M(\mathcal{A})] \\ &= \frac{m_D^2}{M(\mathcal{A})} n_\nu + V[M(\mathcal{A})] \end{aligned} \quad (1)$$

for regions in which nonrelativistic neutrinos dominate [4]

and

$$\begin{aligned} V_{\text{eff}}^{\text{REL}} &= m_\nu(\mathcal{A})^2 \frac{n_\nu}{\langle E_\nu \rangle} + V[M(\mathcal{A})] \\ &= \frac{m_D^4}{\langle E_\nu \rangle M(\mathcal{A})^2} n_\nu + V[M(\mathcal{A})] \end{aligned} \quad (2)$$

for regions where relativistic neutrinos dominate. Here V is the fundamental accelaron potential, m_D is a Dirac neutrino mass and $\langle E_\nu \rangle$ is the average (relativistic) neutrino energy. At the minimum of the potential, the neutrino mass is determined in terms of its density n_ν . This creates neutrino mass dependence on the environment with possible relevance to solar [5] and short-baseline neutrino oscillations [6], as well as the cosmic neutrino background [7]. Interestingly, since the “constants” of the Standard Model depend nontrivially on the scalar neutrino density, $m_\nu(\mathcal{A})$ could induce variations in the fine structure constant of quantum electrodynamics, α , which is the focus of this Letter.

The fine structure constant has been measured in the spectra of distant quasars (QSO) for a number of absorption systems. Early high redshift measurements of α with the Keck telescope found no discrepancy in comparison with laboratory measurements of α to an accuracy of a few parts in 10^{-4} [8] and other observers also subsequently put upper limits on any discrepancy below the 10^{-5} level. However, a discrepancy of $\Delta\alpha/\alpha = -0.57 \pm 0.10 \times 10^{-5}$ was reported in Ref. [9]. Further observations with the Very Large Telescope found no discrepancy at this level [10], but the parameter estimation methods are currently under debate [11]. Even if a discrepancy exists, it is not excluded that an effect may be imitated by a large change of isotope abundances over the last 10 billion years [12]. Thus further observations are mandated to definitively decide whether or not α is truly constant.

It has been proposed that a variation of α could result from the temporal evolution of a quintessence field [13]. However, the model predicts a rather small variation of α from high redshifts to the present unless the quintessence field has unexpectedly undergone a rapid slowing in the recent past.

Here we pursue the implications of mass-varying neutrinos (MaVaNs) on the variation of α over cosmic time

scales. For a class of dependences of $M(\mathcal{A})$ and V a transition in the neutrino phase may occur as the neutrino density varies [14]. We show that this phase transition allows the existence of two distinct stable phases for α .¹

Our paper is organized as follows. In Sec. II we analyze the requirements that a MaVaN phase transition occurs and discuss the conditions under which the model can circumvent hydrodynamic instabilities that may be manifest in the nonrelativistic regime [14, 16]. Then, in Sec. III, we study the corresponding implications for the time variation of the fine structure constant. We summarize in Sec. IV.

II. PHASE TRANSITION IN THE MASS VARIATION OF NEUTRINOS

The stationary points for the potentials in Eqs. (1) and (2) are given by

$$\frac{dV_{\text{eff}}^{\text{NR}}}{d\mathcal{A}} = \left(-\frac{m_D^2 n_\nu}{M^2} + V'(M) \right) \frac{dM}{d\mathcal{A}} = 0 \quad (3)$$

and

$$\frac{dV_{\text{eff}}^{\text{REL}}}{d\mathcal{A}} = \left(-\frac{2m_D^2 n_\nu}{\langle E_\nu \rangle M^3} + V'(M) \right) \frac{dM}{d\mathcal{A}} = 0, \quad (4)$$

where $V'(M) \equiv \partial V(M)/\partial M$ in the two cases. We choose the phase of the singlet neutrino field so that M is real and nonnegative.

To examine the possibilities for a multiphase structure, we note that in string-based discussions masses and couplings are determined by the minima of stabilized moduli [17]. For our purposes, we take this to mean that $M(\mathcal{A}) \simeq M_o[1 + (\mathcal{A} - \mathcal{A}_o)^2/f^2]$ in the vicinity of the stabilization point \mathcal{A}_o , where f is a positive constant. Without loss of generality, we set $\mathcal{A}_o = 0$. With $M_o \neq 0$, we make the assumption: (I) $M(\mathcal{A})$ has a unique stationary point at its absolute minimum M_o . (Although not essential for the discussion, we adopt the simplifying assumption that M is an even function of \mathcal{A} .) As a consequence, both $V_{\text{eff}}^{\text{NR}}$ and $V_{\text{eff}}^{\text{REL}}$ have stationary points at $\mathcal{A} = 0$ where $dM/d\mathcal{A} = 0$. From Eqs. (3) and (4) additional stationary points will exist if the following permits solutions:

$$\left(\frac{M}{M_o} \right)^j \frac{V'(M)}{V'(M_o)} = \frac{n_\nu}{n_{\nu,c}^i}, \quad (5)$$

where $j = 2, 3$ if $i = \text{NR}, \text{REL}$ for the nonrelativistic and relativistic cases, respectively. Here

$$n_{\nu,c}^{\text{NR}} \equiv \frac{M_o^2}{m_D^2} V'(M_o), \quad n_{\nu,c}^{\text{REL}} \equiv \frac{\langle E_\nu \rangle M_o^3}{2m_D^4} V'(M_o). \quad (6)$$

¹ For a completely different mechanism that leads to an abrupt change in α , see Ref. [15].

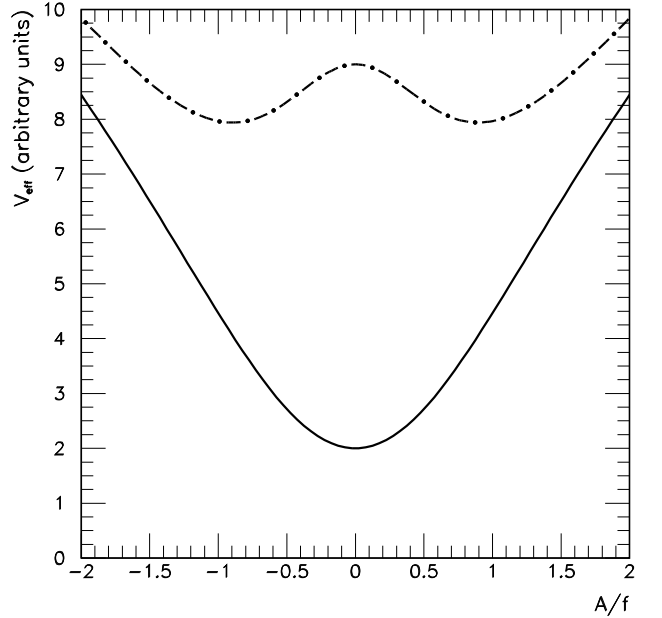


FIG. 1: Qualitative behavior of the effective potential. The dot-dashed line indicates the supercritical regime where $n_\nu > n_{\nu,c}$, whereas the solid line indicates the subcritical regime where $n_\nu < n_{\nu,c}$.

To proceed, rather than examining the system in full generality, we prefer to illustrate the possibilities by imposing a further condition, namely (II) $M^2 V'(M)$ is an increasing function of M . With the help of conditions I and II, Eq. (5) will have solutions in their separate domains if and only if $n_\nu > n_{\nu,c}$. If this condition is fulfilled, the additional stationary points are the two mirror values $\pm \mathcal{A}_{\text{min}}$ corresponding to the value M (assuming there is only one) for which there is a solution. These arguments then imply that the effective potential has the form in Fig. 1, where the lower (upper) curve is valid for $n_\nu < n_{\nu,c}$ ($n_\nu > n_{\nu,c}$).

To illustrate these considerations, and to show how to circumvent the instability issues raised in Ref. [14] in the nonrelativistic regime, we choose a slight variant of the original FNW form [4] for the acceleron potential,

$$V[M(\mathcal{A})] = \Lambda^4 \ln(|M(\mathcal{A})/M_o|), \quad (7)$$

normalized so that $V = 0$ at $\mathcal{A} = 0$. Equation (5) becomes

$$\left(\frac{M}{M_o} \right)^k = \frac{n_\nu}{n_{\nu,c}^i}, \quad (8)$$

where $k = 1, 2$ for $i = \text{NR}, \text{REL}$, respectively. The critical neutrino densities may be calculated using Eq. (6). In terms of the mass of the heaviest neutrino in a dilute

environment ($m_{\nu,0} \equiv m_D^2/M_o \gtrsim \sqrt{\delta m_{\text{atm}}^2}$), we find

$$n_{\nu,c}^{\text{NR}} = \frac{\Lambda^4}{m_{\nu,0}}, \quad n_{\nu,c}^{\text{REL}} = \frac{\langle E_\nu \rangle \Lambda^4}{2 m_{\nu,0}^2} \simeq \frac{T_\nu \Lambda^4}{m_{\nu,0}^2}. \quad (9)$$

Now suppose that for nonrelativistic $\nu + \bar{\nu}$, the neutrino density is subcritical. Then the only stationary point is at $M = M_o$ ($\mathcal{A} = 0$) yielding a neutrino mass $m_\nu = m_{\nu,0}$, which is *independent of the neutrino density*, so that there are *no stability problems*. Instability due to increasing densities can occur at a redshift at which both (a) $n_\nu > n_{\nu,c}$ and (b) the neutrinos are nonrelativistic, with temperature $T_\nu \lesssim 1.2 \Lambda$ [14]. However, recent work [18] has shown that the instability can be avoided for sufficiently weak coupling of the neutrinos to the accelaron during the relevant cosmological era. We explicitly show below that there is indeed a window of late-time instability for the FNW model [4], but that it can be avoided if the accelaron couplings are gravitational or stringy in origin.

In the nonrelativistic regime, and for $n_\nu > n_{\nu,c}^{\text{NR}}$, from Eqs. (8)-(9) we have $m_\nu = \Lambda^4/n_\nu$ with $n_\nu = 3\zeta(3)/(2\pi^2)T_\nu^3$. The neutrinos will be nonrelativistic if

$$\frac{\sqrt{\langle p_\nu^2 \rangle}}{m_\nu} = \sqrt{\frac{15\zeta(5)}{\zeta(3)} \frac{T_\nu n_\nu}{\Lambda^4}} < 1, \quad (10)$$

where $\langle p_\nu^2 \rangle$ is the mean square neutrino momentum in the Fermi-Dirac distribution. This yields $T_\nu \lesssim 1.1\Lambda$, which effectively coincides with the criterion for instabilities in Ref. [14]. With the condition $n_\nu > n_{\nu,c}^{\text{NR}}$, the window of instability is

$$1.8 \left(\frac{\Lambda}{m_{\nu,0}} \right)^{1/3} \lesssim \frac{T_\nu}{\Lambda} \lesssim 1.1. \quad (11)$$

Thus, instabilities may appear if $\Lambda/m_{\nu,0} \lesssim 0.23$.

Since $T_\nu = T_{\nu,0}(1+z)$ (with $T_{\nu,0} \simeq 1.7 \times 10^{-4}$ eV), Eq. (11) can be expressed in terms of redshift as

$$2.9 \left(\frac{\Lambda_{-3}^4}{m_{\nu,0}/0.05 \text{ eV}} \right)^{1/3} \lesssim 1+z \lesssim 6.5 \Lambda_{-3}, \quad (12)$$

where $\Lambda_{-3} \equiv \Lambda/(10^{-3} \text{ eV})$.

Instabilities may be avoided if the coupling between the accelaron and neutrinos β satisfies the inequality [18]

$$\beta \equiv \left| \frac{d \ln m_\nu}{d\mathcal{A}} \right| = \left| \frac{d \ln M}{d\mathcal{A}} \right| < \sqrt{\frac{\Omega_{\text{CDM}} - \Omega_\nu}{2\Omega_\nu}} \frac{1}{M_{\text{Pl}}}, \quad (13)$$

where Ω_{CDM} (Ω_ν) is the cold dark matter (neutrino) dimensionless density. For values of interest, $\beta < 10/M_{\text{Pl}}$ [18]. So far our analysis has been independent of a particular choice of $M(\mathcal{A})$; however, a test of this criterion, and in what follows, the results are somewhat dependent on this choice. We consider two simple forms (which also satisfy assumptions I and II above):

$$M = M_o e^{\mathcal{A}^2/f^2} \quad (14)$$

and

$$M = M_o \cosh \mathcal{A}/f, \quad (15)$$

reminiscent of M-theory potentials for moduli [19]. We analyze the first case in detail, and only provide results for the second. From Eq. (13),

$$\beta = \frac{2|\mathcal{A}|}{f^2} < \frac{10}{M_{\text{Pl}}}. \quad (16)$$

A bound on $|\mathcal{A}|/f$ may be obtained by imposing the nonrelativistic criterion $T_\nu/\Lambda < 1.1$. From Eqs. (8) and (9),

$$e^{\mathcal{A}^2/f^2} = \frac{3\zeta(3)}{2\pi^2} \frac{m_{\nu,0}}{\Lambda} \left(\frac{T_\nu}{\Lambda} \right)^3, \quad (17)$$

which gives

$$\frac{|\mathcal{A}|}{f} < \sqrt{\ln \frac{12 (m_{\nu,0}/0.05 \text{ eV})}{\Lambda_{-3}}}. \quad (18)$$

In our discussion of the α variation we will find that $\Lambda_{-3} \simeq 0.6(m_{\nu,0}/0.05 \text{ eV})^{1/4}$, so that for $m_{\nu,0}$ not much in excess of 0.05 eV, $|\mathcal{A}|/f < 1.7$. From Eq. (16) we see that $f/M_{\text{Pl}} > 0.34$ serves as a sufficient condition to avoid instabilities.

For the alternate $M(\mathcal{A})$ of Eq. (15), we find $\beta = |\tanh(\mathcal{A}/f)|/f \leq 1/f$ for all \mathcal{A} , so that $f/M_{\text{Pl}} > 0.1$ provides a sufficient condition for weak coupling and stability.

III. DISCONTINUITY IN THE FINE STRUCTURE CONSTANT

Allowing for couplings between the accelaron and standard model fields,² the free Lagrangian for the electromagnetic field tensor $F_{\mu\nu}$ can be written as

$$\tilde{\mathcal{L}}_{\text{em}} = -\frac{1}{4} Z_F(\mathcal{A}/M_{\text{Pl}}) F_{\mu\nu} F^{\mu\nu}, \quad (19)$$

which on expansion about the present value \mathcal{A}_0 of \mathcal{A} , becomes

$$\tilde{\mathcal{L}}_{\text{em}} = -\frac{1}{4} (1 + \kappa \Delta\mathcal{A}/M_{\text{Pl}} + \dots) F_{\mu\nu} F^{\mu\nu}, \quad (20)$$

with $\Delta\mathcal{A} = \mathcal{A} - \mathcal{A}_0$ and $\kappa \equiv \partial_{\mathcal{A}} Z_F|_{\mathcal{A}_0}$. The field renormalization $A_\mu \rightarrow A_\mu/Z_F^{1/2}$ to obtain a canonical kinetic energy, generates an effective charge $e/Z_F^{1/2}$. Following

² We do not prescribe a mechanism (which may be desirable for technical naturalness) via which loop corrections to the accelaron potential are suppressed.

Ref. [13], we expand to linear order about the present value e_0 , to obtain

$$\left| \frac{\Delta\alpha}{\alpha} \right| = \kappa \frac{\Delta\mathcal{A}}{M_{\text{Pl}}} = \kappa \frac{\mathcal{A}}{f} \cdot \frac{f}{M_{\text{Pl}}} , \quad (21)$$

where $\alpha \equiv e^2/(4\pi)$. Equation (21) reflects our assumption that the variation in α is uniquely derived from the evolution of the acceleron.

In order to accommodate the meteorite data [20], which do not show evidence for a time-dependent α , we require that \mathcal{A} not vary from ground state equilibrium ($\mathcal{A}_o = 0$) for $z \lesssim 0.5$. Consequently, the model predicts no variation of α during this era, in agreement with existing limits [21].

From Eq. (12), for a transition at $z = 0.5$,

$$\Lambda_{-3} \simeq 0.61(m_{\nu,0}/0.05 \text{ eV})^{1/4} , \quad (22)$$

or equivalently

$$\frac{\rho\mathcal{A}}{\rho_{\text{DE}}} \sim 4 \times 10^{-3} \frac{m_{\nu,0}}{0.05 \text{ eV}} . \quad (23)$$

This precludes Λ saturating the present dark energy. This ratio is roughly similar to the neutrino contribution to the dark matter density. Large scale surveys and WMAP together constrain the neutrino energy density to be $\Omega_\nu \lesssim 0.02$, whereas terrestrial measurements of the neutrino mass indicate $\Omega_\nu > 7 \times 10^{-4}$.

From Eqs. (12) and (22), the density is supercritical and the neutrinos are nonrelativistic for $0.5 < z \lesssim 4$. So there will be a tiny variation of α , in agreement with observations of absorption lines in the spectra of distant QSO [9]. The z -dependence of this variation may be obtained in a straightforward manner. From Eq. (8), at the minimum,

$$\frac{M(\mathcal{A})}{M_o} = \left(\frac{1+z}{1+z_c} \right)^3 , \quad (24)$$

where z_c ($\simeq 0.5$ in our case) is the redshift for which $n_\nu = n_{\nu,c}^{\text{NR}}$. For the gaussian potential, we have

$$\frac{|\mathcal{A}|}{f} = \sqrt{3 \ln \left(\frac{1+z}{1+z_c} \right)} , \quad (25)$$

and for the cosh potential,

$$\frac{|\mathcal{A}|}{f} = \cosh^{-1} \left[\left(\frac{1+z}{1+z_c} \right)^3 \right] . \quad (26)$$

At $z = 2$ (the intermediate point of the data), $|\mathcal{A}|/f \simeq 1.4(2.8)$ for the gaussian and cosh cases, respectively. Taking $f/M_{\text{Pl}} \geq 0.34(0.1)$ for the two cases from the previous section, we find from Eq. (21),

$$\left| \frac{\Delta\alpha}{\alpha} \right| \gtrsim 0.5 \kappa \quad \text{gaussian} \quad (27)$$

$$\left| \frac{\Delta\alpha}{\alpha} \right| \gtrsim 0.3 \kappa \quad \text{cosh} \quad (28)$$

Thus, accommodating the possible variation of α at a level of 5 parts per million requires $\kappa \sim 10^{-5}$.

The quantity κ may be bounded by available limits on $\Delta\alpha/\alpha$ during the eras of recombination ($z \simeq 1100$) and big bang nucleosynthesis (BBN) ($z \sim 10^{10}$). Since neutrinos are relativistic at these redshifts, there are no stability problems. It is straightforward to show that $n_\nu > n_{\nu,c}^{\text{REL}}$ as soon as the neutrinos become relativistic. Then, the system is in the dot-dashed phase of Fig. 1, and at the (mirror) minima the field \mathcal{A} is given by relativistic case of Eq. (8),

$$\frac{M}{M_o} = \sqrt{\frac{3\zeta(3)}{2\pi^2} \frac{T_\nu^2 m_{\nu,0}^2}{\Lambda^4}} , \quad (29)$$

which with Eq. (22) yields

$$|\mathcal{A}|/f \simeq \sqrt{\ln(10z)} \quad \text{gaussian} \quad (30)$$

$$|\mathcal{A}|/f \simeq \cosh^{-1}(10z) \quad \text{cosh} \quad (31)$$

Inserting these in Eq. (21), we find for the recombination and BBN eras,

$$\left| \frac{\Delta\alpha}{\alpha} \right| \simeq \frac{b_{\text{rec}} \kappa f}{M_{\text{Pl}}} , \quad \left| \frac{\Delta\alpha}{\alpha} \right| \simeq \frac{b_{\text{BBN}} \kappa f}{M_{\text{Pl}}} , \quad (32)$$

where $b_{\text{rec}} = 3, 10$ and $b_{\text{BBN}} = 5, 26$ for the gaussian and cosh potentials, respectively. (The slow variation with $m_{\nu,0}$ has been ignored in both cases.) Existing limits on the variation of the fine structure constant, $|\Delta\alpha/\alpha| \lesssim 0.02$ (at the 95% C. L.) for both recombination [22] and BBN [23], in conjunction with the lower bounds on f/M_{Pl} can now be translated into bounds on the coupling constant:

$$\kappa < 0.02 \quad \text{recombination} \quad (33)$$

$$\kappa < 0.01 \quad \text{BBN} , \quad (34)$$

for the gaussian potential and

$$\kappa < 0.02 \quad \text{recombination} \quad (35)$$

$$\kappa < 0.008 \quad \text{BBN} , \quad (36)$$

for the cosh potential. The bounds on κ are a few orders of magnitude larger than the required value to accommodate existing data in the subrelativistic regime.

IV. SUMMARY

In the MaVaN framework, we have constructed a cosmological model that can accommodate limits on the variation of the fine structure constant on short time scales, as well as the potential observation of a variation in α from distant QSOs. The model has a phase transition in the neutrino mass at a redshift $z = 0.5$ that gives a phase transition in α . The existence of this phase transition precludes that the vacuum energy associated

with the acceleration field saturates the present dark energy. To circumvent hydrodynamic instabilities we assumed a sufficiently weak coupling (perhaps stringy in origin) of the neutrinos to the acceleration during the cosmological evolution. The model is consistent with limits on $\Delta\alpha/\alpha$ from recombination and primordial nucleosynthesis.

Acknowledgments

We thank Lily Schrempp for valuable discussions. LA is supported by the University of Wisconsin Milwaukee.

VB is supported by the U.S. Department of Energy (DoE) under Grant No. DE-FG02-95ER40896, and by the Wisconsin Alumni Research Foundation. HG is supported by the U.S. National Science Foundation (NSF) Grant No PHY-0244507. DM is supported by the DoE under Grant No. DE-FG02-04ER41308, and by the NSF under CAREER Award No. PHY-0544278.

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